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Wall shear-stress statistics for the turbulent boundary layer by use of a predictive wall-model with LES

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Abstract

Time-wise velocity signals obtained from large-eddy simulation (LES) within the near-wall, logarithmic region of the zero-pressure gradient, flat-plate turbulent boundary layer are used as input to a calibrated, empirical wall model to calculate the statistics of the fluctuating, wall shear stress τ_w. The results, together with those from numerical simulation and experiment indicate a log-variation of $\overline{(\tau_w'^+)^2}$ with increasing Re_τ.

Introduction

Wall-bounded turbulent flows are well known to contain complex interactions involving several length and velocity scales. An outer flow scale is usually defined as a pipe radius or channel half-thickness for confined flows or a spatially varying thickness for boundary-layer type flows. Corresponding velocity scales are either a bulk velocity or outer free-stream speed. For simple canonical wall-bounded turbulent flows, the inner scale is the viscous scale ν/u_τ where ν is the kinematic viscosity and $u_\tau \equiv \sqrt{\overline{\tau_w}/\rho}$ is the friction or inner velocity scale, where ρ is the fluid density and $\overline{\tau_w}$ is the mean wall shear stress.

It is now widely recognized, however, that there exist streamwise, super length scales, up to $15-20$ times the wall-normal outer length scale that are associated with so-called very large scale motions (VLSMs) active in the outer logarithmic regions of zero- and low-pressure-gradient wall bounded flows [1, 2, 3, 4, 5, 6]. Further, experimental evidence suggests that the dynamical influence of roll-like motions that comprise the VLSMs extends to the very-near wall region where they amplitude modulate near-wall small-scale structures [7, 6].

Marusic *et al.* [8] and Mathis *et al.* [9] built a simple empirical algebraic model, henceforth referred to as the predictive, inner-outer (PIO) model of the effect of the VLSM motions on the near-wall streamwise turbulent fluctuations. The PIO model is driven by an input velocity time-series $u_{OL}(t)$ taken at a fixed wall-normal point $z_o^+ \equiv z_0\,\overline{u_\tau}/\nu$ in the log region of the turbulent wall layer. The output is a corresponding time series of the fluctuating stream-wise velocity at a sequence of wall-normal positions across a profile within the inner part of the layer. Parameters in the model are obtained by a calibrating experiment at a fixed $Re_\tau = \delta\overline{u_\tau}/\nu$ but are considered universal. It is hypothesized that the modulating signal produced by the VLSM is contained within the time-series $u_{OL}(t)$ with possible Re_τ dependence.

In references [8, 9] the PIO model was applied to wall-normal profiles of the stream-wise turbulence intensities over $5 \le z^+ \le 200$ approximately ($z^+ \equiv z\overline{u_\tau}/\nu$), and $2,800 \le Re_\tau \le 19,000$. The model predicts an increase in the peak, near-wall turbulence intensity that agrees well with direct experimental measurement, both consistent with a variation like $\overline{u'^2}/\overline{u_\tau^2} \sim \log(Re_\tau)$. They also applied the model to SLTEST data with consistent results at $Re_\tau \sim O(10^6)$. Inoue *et al.*[10] used the resolved-scale velocity field obtained from LES of the zero pressure gradient, flat-plate turbulent boundary layer (ZPGFPTBL) to supply the $u_{OL}(t)$ time series, thus effectively extending the LES to the near-wall region *a posteriori* for Re_τ in the range $7,000-200,000$. The predicted peak $\overline{u'^2}/\overline{u_\tau^2}$ variation with Re_τ was consistent with a $\log(Re_\tau)$ increase.

Recently Mathis *e*t al.[11] have extended the PIO model to the construction of a time series of the instantaneous $\tau_w(t)$ itself, for the ZPGFPTBL. They demonstrate that the extended model, also driven by the $u_{OL}(t)$ time series, is capable of successfully capturing the Re_τ variation of the variance of the fluctuating wall-shear stress, $\overline{(\tau_w'^+)^2}$ as observed in both direct numerical simulation (DNS) and other experimental studies. The purpose of the present paper is to apply LES to this statistic at Re_τ values out of the range of present experiments. In what follows, we first briefly describe the PIO model for the wall shear-stress fluctuation. This is followed by a brief description of the LES performed and the results obtained.

Predictive Inner-Outer Model

The aim is to investigate the statistics of the fluctuating component of the wall shear stress

$$\tau_w'(x,y,t) \equiv \tau_w(x,y,t) - \overline{\tau_w(x,y)}, \qquad (1)$$

where $\tau_w(x,y,t)$ is the instantaneous wall shear stress at position (x,y) on the wall and with temporal mean $\overline{\tau_w(x,y)}$. The flow is the ZPGFPTBL. The semi-empirical wall model is the PIO [8, 9] extended into the viscous sub-layer where the model takes the form of a linear relationship between $u_{OL}^+(t) \equiv u_{OL}^+(t)/\overline{u_\tau}$ and the instantaneous wall-shear stress. Mathis *e*t al. [11] formulate the model as

$$\tau_{wp}'^+ = \tau_w'^*(t^+)\{1+\alpha u_{OL}'^+)(t^+)\} + \alpha u_{OL}'^+(t^+), \qquad (2)$$

where $\tau_{wp}'^+ \equiv \tau_{wp}'^+/(\rho\,\overline{u_\tau^2})$ is the predicted, fluctuating wall shear stress normalized with inner variables, $t^+ = t\,u_\tau^2/\nu$. The time series $\tau_w'^*(t^+)$ and the parameter α are determined by a suitable calibration experiment [11] and are hypothesized to be Re_τ independent. The first part of (2) models amplitude modulation of the small-scales by the VLSM as contained in $u_{OL}'^+(t^+)$ while the second term models the superposition of the long scales themselves as felt right at the wall. In the calibrating experiment, the signal $u_{OL}'^+(t^+)$ is obtained at a wall-normal distance $z_O^+ = \sqrt{15\,Re_\tau}$ which is taken as the near geometric center of the log layer.

Large Eddy Simulation (LES)

The LES and the velocity time series obtained from the LES are the same as those described in Inoue *e*t al.[10] and used therein, together with the PIO model for the prediction of stream-wise turbulent intensities. It utilizes the approach of [12], extended to the smooth-wall ZPGTBL in [13]. The method combines the stretched-vortex SGS model for outer-flow LES, with a tailored wall model. The near-wall treatment has two parts. First, an analytic integration of the stream-wise momentum equation across a constant-thickness, wall-adjacent layer is performed. Using inner-scaling to reduce the unsteady term, this leads to an ordinary-differential equation (ODE) in time, at each point on the wall, for the fluctuating (but effectively filtered) friction velocity $u_\tau(x,y,t)$ where (x,y,z) are stream-wise, span-wise and wall-normal co-ordinates respectively. Each ODE contains inertial and viscous source terms that can be obtained from the LES, thus coupling the ODE to the outer LES while providing dynamical computation of the local wall shear stress $\tau_w = \rho\, u_\tau^2$. Second, the stretched vortex SGS model is used within the near-wall region where it is assumed that the subgrid vortex elements represent attached vortices in the presence of the wall, with assumed scaling linearly proportional to distance from the wall. This is intended to capture the principal dynamical behavior of longitudinal vortices in wall-normal transport of stream-wise momentum. Wall-normal integration then leads to a “log-like” relationship for the mean SGS stream-wise velocity in an inner, modeled layer bounded above by a “virtual wall” at $z = h_0$ and below by the actual no-slip wall $z = 0$. The log-relation contains u_τ (obtained from the ODE) and provides a slip-velocity boundary condition for the outer-flow LES at $z = h_0$. In this wall-model both $u_\tau(x,y,t)$ and an effective Kármán constant are calculated dynamically.

There are two adjustable parameters, first the height of the virtual wall $h_0/\Delta z$, where Δz is the (constant) wall-normal cell size, and second, an intercept $h_\nu^+ = 10.23$ between the standard log-profile and the linear, buffer-layer velocity profile. The latter is strictly for smooth wall flow and is the only empirical parameter in the composite wall-model/LES. LES of channel flow at Re_τ up to 2×10^7 were reported in [12]. In [13] this model was applied to LES of the smooth-wall, spatially developing, zero-pressure gradient turbulent boundary layer in the the range $Re_\theta = 10^3 - 10^{12}$, and details of the present numerical algorithm may be found therein.

The six individual LES cases from which the present results were obtained are summarized in table 1 where it can be seen that Re_τ is in the range $6.89\times 10^3 - 2.00\times 10^5$. For all cases the domain size was $(L_x/\delta_0, L_y/\delta_0, L_z/\delta_0) = (72,6,4)$ where δ_0 is the boundary-layer thickness at the domain inlet. For each LES run, a span-wise rake of velocity-time series was obtained at $z^+ \approx z_O^+ \equiv \sqrt{15\,Re_\tau}$. The velocity signal was taken from $x_{\mathrm{stat}} = 0.75L_x$ except for Case 4, where $x_{\mathrm{stat}} = 0.45L_x$. When normalized by the time averaged $\overline{u_\tau}$, the LES velocity-time series provides the non-dimensional signal input u_{OL}^+ to the PIO wall model. Equation (2) then supplies a time series of predicted wall shear stress fluctuations $\tau_{wp}'^+$ over the non-dimensional time period of duration $T^+ \equiv T\,u_\tau^2/\nu$ shown in table 1 for each case. Our largest Re_τ is set by the requirement that z_O^+ must lie within the first few wall-normal grid points coupled to the result $z_0/\delta_{99} \sim Re_\tau^{-1/2}$, so that z_0 must move relatively closer to the wall as Re_τ increases.

Results and discussion

Figure 1, reproduced from [10] shows mean velocity profiles for all present cases $1-6$ of Table 1. The mean velocity clearly show the wake and log-like regions. The slight drop in each profile close to the wall is a wall-effect discussed in [13]. In the figure, the symbol The symbol "$\times$" marks the location corresponding to $z_O^+ \equiv \sqrt{15\,Re_\tau}$ where the velocity time series component is measured for each case. It can be seen that this in general does not correspond to a wall normal cell location where the stream-wise component of the resolved-scale is stored. Hence fourth-order, wall-normal interpolation was used at each time instant to calculate the actual u_{OL}.

From the individual time series produced from application of 2, any required one-point statistics of τ_w' can be calculated at the wall location below which the time series is captured. Figure 2 shows calculated values of the variance of the fluctuating wall shear stress calculated as

$$\overline{(\tau_w'^+)^2} = \frac{\overline{(\tau_w')^2}}{(\overline{\tau_w})^2}. \tag{3}$$

Also shown in the figure are results from DNS studies at lower Re_τ [14, 15] and also those obtained by applying the PIO model to time series obtained experimentally [11]. Of particular note is the single point obtained from SLTEST data at $Re_\tau = O(10^6)$. Taken together, the DNS results, the experimentally-based predictions and the LES-based results indicate a slow increase in $\overline{(\tau_w'^+)^2}$ with Re_τ. This appears to be consistent with a log-like increase. Hence the solid line is a curve-fit of the form

$$\overline{(\tau_w'^+)^2} = 0.085 + 0.013\,\ln(Re_\tau) \tag{4}$$

which appears to fit the composite results quite well.

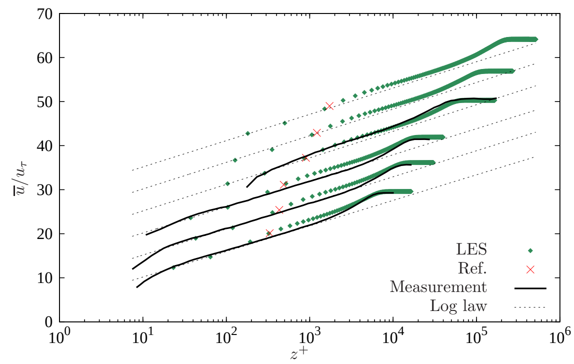

Figure 1: Mean velocity profile for cases $1-6$ (from bottom to top). The experimental measurements at $Re_\tau = 7{,}300$, $13{,}600$ and $19{,}000$ are from Mathis *et al.* [9], and that at $Re_\tau = 62{,}000$ is from Winkel *et al.* [16]. Profiles are displaced 5 units of $\overline{u}/u_\tau$ for clarity. Dashed-lines are log law using dynamically calculated Kármán “constant”.

Conclusions

Time-wise velocity signals from within the near-wall, logarithmic region of the zero-pressure gradient, flat-plate turbulent boundary layer, are obtained from large-eddy simulation (LES). These are then used as input to an empirical wall model [11] to calculate the statistics of the fluctuating, wall shear stress τ_w. The wall model is based on the idea that near-wall, small-scale motions show universal, Reynolds-number independent behavior but are modulated by large or very large-scale motion within the logarithmic region, that themselves contain Reynolds number dependence. Hence a log-region time series that captures these events, in combination with a calibration experiment at one Reynolds number to capture universal parameters, is sufficient to predict the Reynolds-number dependence of the

LES	N_x	N_y	N_z	T^+	$Re_{\tau,99}$
1	768	64	128	1.44×10^6	6.89×10^3
2	768	64	128	2.07×10^6	1.25×10^4
3	1152	96	192	1.41×10^6	1.60×10^4
4	1728	144	288	8.72×10^5	5.44×10^4
5	2304	192	384	1.21×10^6	9.96×10^4
6	3072	256	512	1.00×10^6	2.00×10^5

Table 1: LES cases showing resolution $N_x \times N_y \times N_z$, dimensionless time over which time series was taken $T^+ = T\,u_\tau^2/\nu$ and Reynolds number $Re_{\tau,99}$ based on the 99% boundary layer thickness δ_{99}.

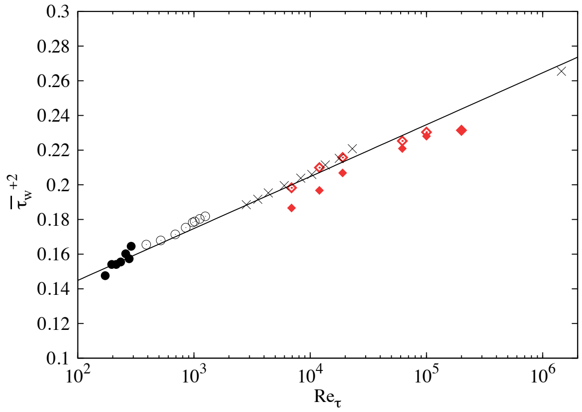

Figure 2: $\overline{(\tau_w'^+)^2}$ versus $Re_\tau \equiv u_\tau \delta/\nu$. Shown are results obtained from direct numerical simulation (DNS), predictions using both experimentally measured and LES-generated time series. Filled and open circles are DNS results respectively from [14] and [15]; Crosses indicate results from the model using experimental data from the log region. Red-coloured diamonds are from the model using LES data from the log region (filled and unfilled symbols indicate different filter sizes). Solid line is $0.085 + 0.013\ln(Re_\tau)$.

wall shear-stress statistics. The DNS, experimentally-based and LES-predictions are broadly consistent with a log-like increase of $\overline{(\tau_w'^+)^2}$ with Re_τ. This result, over a wide range of Re_τ, provides further support for the hypothesis that the dynamical effects of large-scale motions that are present in the log region of the outer layer, play an active role right at the wall. The present LES appears capable of capturing these motions on long domains, up to $Re_\tau = 2 \times 10^5$.

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